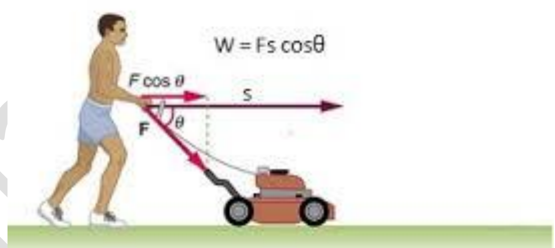
Work- Work done *W* is defined as the dot product of force *F* and displacement *s*.

$$W = \vec{F}.\vec{s}$$
$$= Fs\cos\theta$$

Here ϑ is the angle between $\vec{F}$ and $\vec{s}$.

Work done by the force is positive if the angle between force and displacement is acute ($0^\circ<\theta<90^\circ$) as cos ϑ is positive. This signifies, when the force and displacement are in same direction, work done is positive. This work is said to be done upon the body.

When the force acts in a direction at right angle to the direction of displacement ($\cos 90^\circ = 0$), no work is done (zero work).



Work done by the force is negative if the angle between force and displacement is obtuse ($90^\circ<\theta<180^\circ$) as cos$\vartheta$ is negative. This signifies, when the force and displacement are in opposite direction, work done is negative. This work is said to be done by the body.

Work done by a variable force:-

If applied force *F* is not a constant force, then work done by this force in moving the body from position *A* to *B* will be,

$$W = \int_A^B \vec{F}.d\vec{s}$$

Here *ds* is the small displacement.

Units: The unit of work done in S.I is joule (J) and in C.G.S system is erg.

1J = 1 N.m , 1 erg = 1 dyn.cm

Relation between Joule and erg:- 1 J = 10^7 erg

Power:-The rate at which work is done is called power and is defined as

P = W/t = F.s/v = F.v

Here *s* is the distance and *v* is the speed.

Instantaneous power in terms of mechanical energy:- $P = dE/dt$

Units: The unit of power in S.I system is *J/s* (watt) and in C.G.S system is *erg/s*.

Energy:-

1) Energy is the ability of the body to do some work. The unit of energy is same as that of work.

2) Kinetic Energy (K):- It is defined as, = ½ mv^2

Here *m* is the mass of the body and *v* is the speed of the body.

Potential Energy (*U*):- Potential energy of a body is defined as, *U = mgh* Here, m is the mass of the body, g is the free fall acceleration acceleration due to gravity) and h is the height.

Gravitational Potential Energy:- An object's gravitational potential energy *U* is its mass *m* times the acceleration due to gravity *g* times its height *h* above a zero level.

In symbol's,

U = mgh

Relation between Kinetic Energy (*K*) and momentum (*p*):-

$K = p^2/2m$

If two bodies of different masses have same momentum, body with a greater mass shall have lesser kinetic energy.

If two bodies of different mass have same kinetic energy, body with a greater mass shall have greater momentum.

For two bodies having same mass, the body having greater momentum shall have greater kinetic energy.

Work energy Theorem:- It states that work done on the body or by the body is equal to the net change in its kinetic energy .

For constant force,

$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$

= Final K.E – Initial K.E

For variable force,

$$W = \int_A^B F\, ds$$

? $$= \frac{1}{2} m{v_B}^2 - \frac{1}{2} m{v_A}^2$$

Law of conservation of energy:- It states that, "Energy can neither be created nor destroyed. It can be converted from one form to another. The sum of total energy, in this universe, is always same".

The sum of the kinetic and potential energies of an object is called mechanical energy. So, $E = K+U$

In accordance to law of conservation of energy, the total mechanical energy of the system always remains constant.

So, $mgh + \frac{1}{2} mv^2$ = constant

In an isolated system, the total energy E_{total} of the system is constant.

So, $E = U+K$ = constant

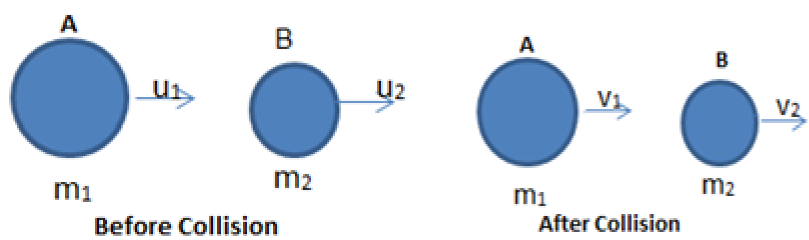
Or, $U_i+K_i = U_f+K_f$

Or, ?U = -?K

Speed of particle v in a central force field:

$v = \sqrt{2/m}\ [E-U(x)]$

Conservation of linear momentum:-



? In an isolated system (no external force ($F_{ext} = 0$)), the total momentum of the system before collision would be equal to total momentum of the system after collision. So, $p_f = p_i$

Coefficient of restitution *(e)*:- It is defined as the ratio between magnitude of impulse during period of restitution to that during period of deformation.

e = relative velocity after collision / relative velocity before collision

$= v_2 - v_1/u_1 - u_2$

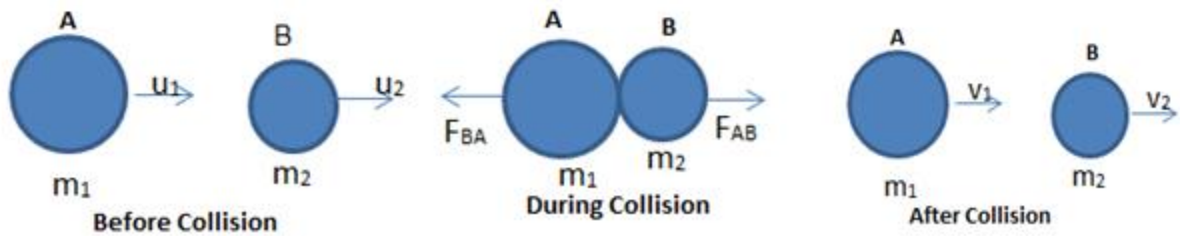
Case (i) For perfectly elastic collision, $e = 1$. Thus, $v_2 - v_1 = u_1 - u_2$. This signifies the relative velocities of two bodies before and after collision are same.

Case (ii) For inelastic collision, e<1. Thus, $v_2 - v_1 < u_1 - u_2$. This signifies, the value of e shall depend upon the extent of loss of kinetic energy during collision.

Case (iii) For perfectly inelastic collision, $e = 0$. Thus, $v_2 - v_1 = 0$, or $v_2 = v_1$. This signifies the two bodies shall move together with same velocity. Therefore, there shall be no separation between them.

Elastic collision:- In an elastic collision, both the momentum and kinetic energy conserved.

One dimensional elastic collision:-?



After collision, the velocity of two body will be**,**

$v_1 = (m_1\text{-}m_2/\ m_1+m_2)u_1 + (2m_2/\ m_1+m_2)u_2$ and $v_2 = (m_2\text{-}m_1/\ m_1+m_2)u_2 + (2m_1/\ m_1+m_2)u_1$

Case:I

When both the colliding bodies are of the same mass, i.e., $m_1 = m_2$, then,

$v_1 = u_2$ and $v_2 = u_1$

Case:II

When the body *B* of mass m_2 is initially at rest, i.e., $u_2 = 0$, then,

$v_1 = (m_1\text{-}m_2/\ m_1+m_2)u_1$ and $v_2 = (2m_1/\ m_1+m_2)u_1$

(a) When $m_2 << m_1$, then, $v_1 = u_1$ and $v_2 = 2u_1$

(b) When $m_2 = m_1$, then, $v_1 = 0$ and $v_2 = u_1$

(c) When $m_2 >> m_1$, then, $v_1 = -u_1$ and v_2 will be very small.

Inelastic collision:- In an inelastic collision, only the quantity momentum is conserved but not kinetic energy.

$v = (m_1u_1+m_2u_2)/(m_1+m_2)$

and

loss in kinetic energy, $E = ½\, m_1u_1^2 + ½\, m_2u_2^2 - ½\, (m_1 + m_2)v^2$

or,

$E = ½\, (m_1u_1^2 + m_2u_2^2) – ½\, [(m_1u_1 + m_2u_2)/(m_1 + m_2)]^2$

$= m_1\, m_2\, (u_1 - u_2)^2 / 2(m_1 + m_2)$

Points to be Notice:-

(i) The maximum transfer energy occurs if $m_1 = m_2$

(ii) If K_i is the initial kinetic energy and K_f is the final kinetic energy of mass m_1, the fractional decrease in kinetic energy is given by,

$K_i – K_f / K_i = 1 - v_1^2/u^2_1$

Further, if $m_2 = nm_1$ and $u_2 = 0$, then,

$K_i – K_f / K_i = 4n/(1+n)^2$

Conservation Equation:

(i) Momentum – $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(ii) Energy – $½\, m_1u_1^2 + ½\, m_2u_2^2 = ½\, m_1v_1^2 + ½\, m_2v_2^2$

Conservative force (*F*):- Conservative force is equal to the negative gradient of potential *V* of the field of that force. This force is also called central force.

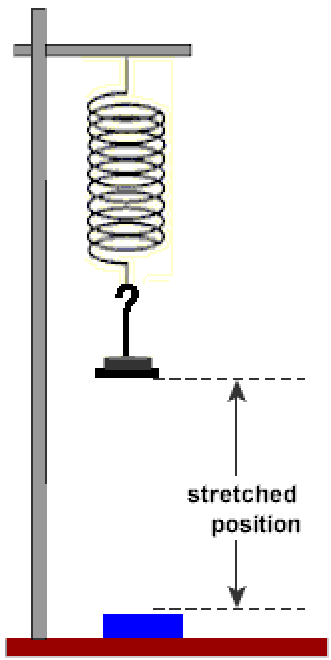
So, $F = -(dV/dr)$

The line integral of a conservative force around a closed path is always zero.

So,

$$\oint \vec{F}.d\vec{r} = 0$$

Spring potential energy (E_s):- It is defined as,



$E_s = ½ kx^2$

Here k is the spring constant and x is the elongation.

Equilibrium Conditions:

(a) Condition for equilibrium, $dU/dx = 0$

(b) For stable equilibrium,

$U(x)$ = minimum,

$dU/dx = 0$,

d^2U/dx^2 = +ve

(c) For unstable equilibrium,

$U(x)$ = maximum

$dU/dx = 0$

d^2U/dx^2 = -ve

(d) For neutral equilibrium,

$U(x)$ = constant

$dU/dx = 0$

$d^2U/dx^2 = 0$

UNITS AND DIMENSIONS OF WORK, POWER AND ENERGY

Work and Energy are measured in the same units. Power, being the rate at which work is done, is measured in a different unit.

Quantity and Units/Dimensions		Work (Energy)	Power
Dimension		ML^2T^{-2}	ML^2T^{-3}
Absolute	MKS	Joule	Watt
	FPS	ft-Poundal	ft-poundal/sec
	CGS	erg	Erg/sec.
Gravitational	MKS	kg-m	Kg-m/sec
	FPS	ft-lb	ft-lb/sec.
	CGS	gm-cm	Gm-cm/sec
Practical (Other)		kwh, eV, cal	HP, kW, MW

Conversions between Different Systems of Units

1 Joule = 1 Newton ´ 1 m = 10^5 dyne ´ 10^2 cm = 10^7 erg

1 watt = 1 Joule/ sec = 10^7 erg/sec.

1 kwh = 10^3 watt ´ 1 hr = 10^3 watt ´ 3600 sec

= 3.6 ´ 10^6 Joule

1HP = 746 watt.

1 MW = 10^6 watt.

1 cal = 1 calorie = 4.2 Joule

1eV = "e" Joule = 1.6×10^{-19} Joule

(e = magnitude of charge on the electron in coulombs)