- **Sine rule:** Sides of a triangle are proportional to the sine of the angles opposite to them. So, in ΔABC,

sin A/a = sin B/b = sin C/c = 2Δ/abc.

This may also be written as (a/sin A) = (b/sin B) = (c/sin C)

- **Cosine rule:** In any ΔABC,

  cos A = $(b^2 + c^2 – a^2)$ /2bc

  cos B = $(a^2 + c^2 – b^2)$/2ac

  cos C = $(a^2 + b^2 - c^2)$/2ab

- **Trigonometric ratios of half-angles**:

  sin A/2 = √[(s-b)(s-c)/bc]

  sin B/2 = √[(s-c) (s-a)/ac]

  sin C/2 = √[(s-a) (s-b)/ab]

  cos A/2 = √s(s - a)/bc

  cos B/2 = √s(s - b)/ac

  cos C/2 = √s(s - c)/ab

  tan A/2 = √[(s - b) (s - c)/s(s - a)]

  tan B/2 = √[(s - c) (s - a)/s(s - b)]

  tan C/2 = √[(s - a) (s - b)/s(s - c)]

- **Projection rule**: In any ΔABC,

  a = b cos C + c cos Bb = c cos A + a cos C

  c = a cos B + b cos A

- **Area of a triangle**

If Δ denotes the area of the triangle ABC, then it can be calculated in any of the following forms:

$\Delta = 1/2\ bc \sin A = 1/2\ ca \sin B = 1/2\ ab \sin C$

$\Delta = \sqrt{s(s - a)(s - b)(s - c)}$

$\Delta = 1/2. (a^2 \sin B \sin C)/ \sin(B + C)$

$= 1/2. (b^2 \sin C \sin A)/ \sin (C + A)$

$= 1/2. (c^2 \sin A \sin B)/ \sin (A + B)$

- **Semi-perimeter of the triangle**

If S denotes the perimeter of the triangle ABC, then $s = (a + b + c)/2$

- **Napier's analogy**

In any ΔABC,

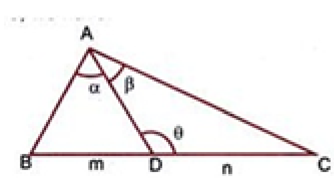
$\tan [(B - C)/2] = (b - c)/(b + c) \cot A/2$

$\tan [(C - A) /2] = (c - a)/(c + a) \cot B/2$

$\tan [(A - B) /2] = (a - b)/(a + b) \cot C/2$

- **m-n theorem**

Consider a triangle ABC where D is a point on side BC such that it divides the side BC in the ratio m: n, then as hown in the figure, the following results hold good:



$(m + n) \cot \theta = m \cot \alpha - n \cot \beta$.

$(m + n) \cot \theta = n \cot B - m \cot C$.

- **Apollonius theorem**

In a triangle ABC, if AD is the median through A, then

$AB^2 + AC^2 = 2(AD^2 + BD^2)$.

- If the three sides say a, b and c of a triangle are given, then angle A is obtained with the help of the formula

  $\tan A/2 = \sqrt{(s - b)(s - c) / s(s - a)}$ or $\cos A = b^2 + c^2 - a^2 / 2bc$.

  Angles B and C can also be obtained in the same way.

- If two sides b and c and the included angle A are given, then

  $\tan (B - C)/2 = (b - c)/(b + c) \cot A/2$

  This gives the value of (B- C)/2.

  Hence, using $(B + C)/2 = 90^o - A/2$ along with the last equation both B and C can be evaluated. Now, the sides can be evaluated using the formula

  $a = b \sin A/\sin B$ or $a^2 = b^2 + c^2 - 2bc \cos A$.

- If two sides b and c and the angle B (opposite to side b) are given, then using the following results, we can easily obtain the remaining elements
  - $\sin C = c/b \sin B$, $A = 180^o - (B + C)$ and $b = b \sin A/\sin B$

- Some of the important points which must be remembered include:

  a. $b < c \sin B$, there is no triangle possible (Fig. 1)
  b. If $b = c \sin B$ and B is an acute angle, then only one triangle is possible (Fig. 2)
  c. If $c \sin B < b < c$ and B is an acute angle, then there are two values of angle C (Fig. 3).
  d. If $c < b$ and B is an acute angle, then there is only one triangle (Fig. 4).

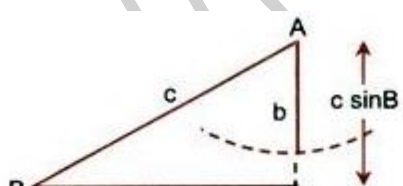


Fig.1

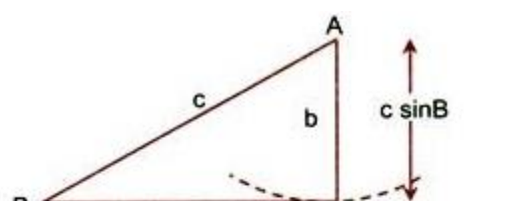


Fig.2

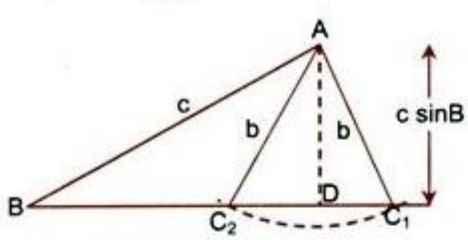


Fig.3

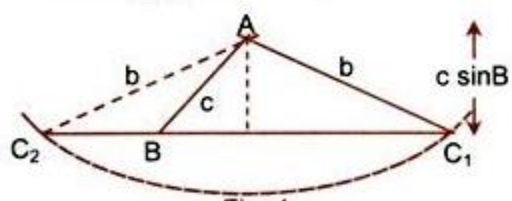


Fig. 4