- In order to solve a quadratic equation of the form $ax^2 + bx + c$, we first need to calculate the discriminant with the help of the formula $D = b^2 – 4ac$.
- The solution of the quadratic equation $ax^2 + bx + c= 0$ is given by $x = [-b \pm \sqrt{b^2 – 4ac}] / 2a$
- If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then we have the following results for the sum and product of roots:
  1. $\alpha + \beta = -b/a$
  2. $\alpha.\beta = c/a$
  3. $\alpha – \beta = \sqrt{D}/a$
- It is not possible for a quadratic equation to have three different roots and if in any case it happens, then the equation becomes an identity.

Nature of Roots:

- Consider an equation $ax^2 + bx + c = 0$, where a, b and c ∈ R and a ≠ 0, then we have the following cases:
  1. D > 0 iff the roots are real and distinct i.e. the roots are unequal
  2. D = 0 iff the roots are real and coincident i.e. equal
  3. D < 0 iffthe roots are imaginary
  4. The imaginary roots always occur in pairs i.e. if a+ib is one root of a quadratic equation, then the other root must be the conjugate i.e. a-ib, where a, b ∈ R and $i = \sqrt{-1}$.
- Consider an equation $ax^2 + bx + c = 0$, where a, b and c ∈Q and a ≠ 0, then
  1. If D > 0 and is also a perfect square then the roots are rational and unequal.
  2. If $\alpha = p + \sqrt{q}$ is a root of the equation, where 'p' is rational and $\sqrt{q}$ is a surd, then the other root must be the conjugate of it i.e. $\beta = p - \sqrt{q}$ and vice versa.
- If the roots of the quadratic equation are known, then the quadratic equation may be constructed with the help of the formula

  x^2 – (Sum of roots)x + (Product of roots) = 0.

  So if α and β are the roots of equation then the quadratic equation is

  $x^2 – (\alpha + \beta)x + \alpha \beta = 0$
- For the quadratic expressiony = $ax^2 + bx + c$, where a, b, c ∈ R and a ≠ 0, then the graph between x and y is always a parabola.
  1. If a > 0, then the shape of the parabola is concave upwards
  2. If a < 0, then the shape of the parabola is concave upwards
- Inequalities of the form P(x)/ Q(x) > 0 can be easily solved by the method of intervals of number line rule.
- The maximum and minimum values of the expression $y = ax^2 + bx + c$ occur at the point x = -b/2a depending on whether a > 0 or a< 0.
  1. $y \in [(4ac-b^2) / 4a, \infty]$ if a > 0
  2. If a < 0, then $y \in [-\infty, (4ac-b^2) / 4a]$
- The quadratic function of the form $f(x, y) = ax^2+by^2 + 2hxy + 2gx + 2fy + c = 0$ can be resolved into two linear factors provided it satisfies the following condition: $abc + 2fgh –af^2 – bg^2 – ch^2 = 0$
- In general, if $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the roots of the equation

  $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, then

1.$\Sigma\alpha_1 = - a_1/a_0$

2.$\Sigma\ \alpha_1\alpha_2 = a_2/a_0$

3.$\Sigma\ \alpha_1\alpha_2\alpha_3 = - a_3/a_0$

$\Sigma\ \alpha_1\alpha_2\alpha_3 \ldots\ldots\alpha_n = (-1)^n\ a_n/a_0$

- Every equation of n^{th} degree has exactly n roots ($n \geq 1$) and if it has more than n roots then the equation becomes an identity.
- If there are two real numbers 'a' and 'b' such that f(a) and f(b) are of opposite signs, then f(x) = 0 must have at least one real root between 'a' and 'b'.
- Every equation f(x) = 0 of odd degree has at least one real root of a sign opposite to that of its last term.