

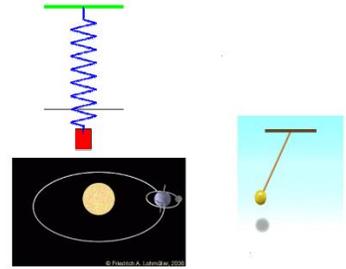
## Types of Motion:-

**(a) Periodic motion:-** When a body or a moving particle repeats its motion along a definite path after regular intervals of time, its motion is said to be Periodic Motion and interval of time is called time or harmonic motion period ( $T$ ). The path of periodic motion may be linear, circular, elliptical or any other curve.

**(b) Oscillatory motion:-** To and Fro' type of motion is called an Oscillatory Motion. It need not be periodic and need not have fixed extreme positions. The force/torque acting in oscillatory motion (directed towards equilibrium point) is called restoring force/tor

**(c) Simple harmonic motion (SHM):-** Simple harmonic motion is the motion in which the restoring force is proportional to displacement from the mean position and opposes its increase.

**Simple harmonic motion (SHM):-** A particle is said to move in SHM, if its acceleration is proportional to the displacement and is always directed towards the mean position.



## Conditions of Simple Harmonic Motion

For SHM to occur, three conditions must be satisfied.

(a) There must be a position of **stable equilibrium**

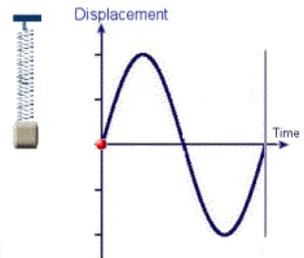
At the stable equilibrium potential energy is minimum.

So,  $dU/dy = 0$  and  $d^2U/dy^2 > 0$

(b) There must be no dissipation of energy

(c) The acceleration is proportional to the displacement and opposite in direction

That is,  $a = -\omega^2 y$



## Equation of SHM:-

(a)  $F = -ky$  (b)  $d^2y/dt^2 + \omega^2 y = 0$

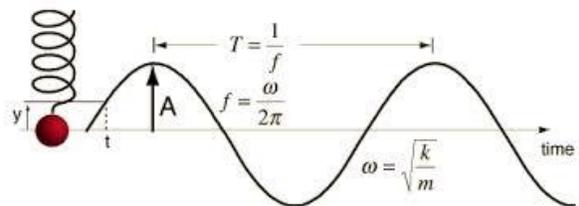
Here  $\omega = \sqrt{k/m}$  ( $k$  is force constant)

## Displacement ( $y$ ) :-

Displacement of a particle vibrating in SHM, at any instant, is defined as its distance from the mean position at that instant.

$$y = r \sin (\omega t + \phi)$$

Here  $\phi$  is the phase and  $r$  is the radius of the circle.



**Condition:**

When,  $\phi = 0$ , then,  $y = r \sin \omega t$

and

When,  $\phi = \pi/2$ , then,  $y = r \cos \omega t$

**Amplitude (r):-**

Amplitude of a particle, vibrating in SHM, is defined as its maximum displacement on either side of mean position.

As the extreme value of value of  $\omega t = \pm 1$ , thus,  $y = \pm r$

**Velocity (V):-**  $V = dy/dt = r\omega \cos(\omega t + \phi) = v \cos(\omega t + \phi) = \omega \sqrt{r^2 - y^2}$

Here v is the linear velocity of the particle.

**Condition:-** When,  $y = 0$ , then,  $V = v = r\omega$

and

When,  $y = \pm r$ , then,  $V=0$

A particle vibrating in SHM, passes with maximum velocity through the mean position and is at rest at the extreme positions.

$$y^2/r^2 + y^2/\omega^2 r^2 = 1$$

Acceleration (a):  $a = dV/dt = (-v^2/r) \sin \omega t = -\omega^2 y$

**Condition:-**

When,  $y = 0$ , then,  $a = 0$

And,

When,  $y = \pm r$ , then,  $a = \pm \omega^2 r$

A particle vibrating in SHM, has zero acceleration while passing through mean position and has maximum acceleration while at extreme positions.

(i) Acceleration is directly proportional to y (displacement).

(ii) Acceleration is always directed towards the mean position.

**Time period (T):-** It is the time taken by the particle to complete one vibration.

$$(a) T = 2\pi/\omega$$

$$(b) T = 2\pi v(\text{displacement}/\text{acceleration})$$

$$(c) T = 2\pi\sqrt{m/k}$$

**Frequency (f):-** It is the number of vibrations made by the body in one second.

$$(a) f = 1/T$$

$$(b) f = 1/2\pi\sqrt{k/m}$$

**Angular frequency ( $\omega$ ):-**

$$(a) \omega = 2\pi/T$$

$$(b) \omega = \sqrt{\text{acceleration}/\text{displacement}}$$

**Relation between Angular frequency ( $\omega$ ) and Frequency (f):-**  $\omega = 2\pi f = \sqrt{k/m}$

**Phase:-**

(a) Phase of a particle is defined as its state as regards its position and direction of motion.

(b) It is measured by the fraction of time period that has elapsed since the particle crossed its mean position, last, in the positive direction.

(c) Phase can also be measured in terms of the angle, expressed as a fraction of  $2\pi$  radian, traversed by the radius vector of the circle of reference while the initial position of the radius vector is taken to be that which corresponds to the instant when the particle in SHM is about to cross mean position in positive direction.

**Energy in SHM:-**

**(a) Kinetic Energy ( $E_k$ ):-**

$$E_k = \frac{1}{2} m\omega^2(r^2 - y^2) = \frac{1}{2} m\omega^2 r^2 \cos^2 \omega t$$

$$\text{When, } y = 0, \text{ then, } (E_k)_{\max} = \frac{1}{2} m\omega^2 r^2 \quad (\text{maximum})$$

And

When,  $y = \pm r$ , then,  $(E_k)_{\min} = 0$  (minimum)

**(b) Potential Energy ( $E_p$ ):-**

$$E_p = \frac{1}{2} m\omega^2 r^2 = \frac{1}{2} m\omega^2 r^2 \sin^2 \omega t$$

$$(E_p)_{\max} = \frac{1}{2} m\omega^2 r^2$$

**(c) Total Energy ( $E$ ):-**

$$E = E_k + E_p = \frac{1}{2} m\omega^2 r^2 = \text{conserved}$$

$$E = (E_k)_{\max} = (E_p)_{\max}$$

**Average Kinetic Energy:**  $\langle E_k \rangle = \left(\frac{1}{4}\right) m\omega^2 r^2$

**Average Potential Energy:**  $\langle E_p \rangle = \left(\frac{1}{4}\right) m\omega^2 r^2$

$$\langle E/2 \rangle = \langle E_k \rangle = \langle E_p \rangle$$

**Spring-mass system:-**

(a)  $mg = kx_0$

(b) Time period,  $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{x_0/g}$

**Massive spring:-** Time period,  $T = 2\pi\sqrt{[m + (m_s/3)]/k}$

**Cutting a spring:-**

(a) Time period,  $T' = T_0/\sqrt{n}$

(b) Frequency,  $f' = \sqrt{n} f_0$

(c) Spring constant,  $k' = nk$

(d) If spring is cut into two pieces of length  $l_1$  and  $l_2$  such that,  $l_1 = nl_2$ , then,

$$k_1 = (n+1/n)k,$$

$$k_2 = (n+1)k$$

and

$$k_1 l_1 = k_2 l_2$$

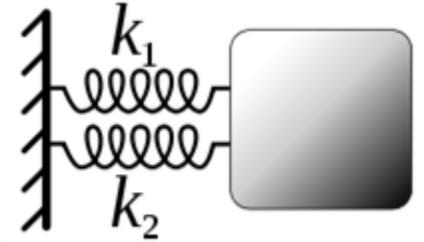
### Spring in parallel connection:-

(a) Total spring constant,  $k = k_1 + k_2$

(b) Time period,  $T = 2\pi\sqrt{m/(k_1+k_2)}$

(c) If  $T_1 = 2\pi\sqrt{m/k_1}$  and  $T_2 = 2\pi\sqrt{m/k_2}$ , then,

$$T = T_1 T_2 / \sqrt{T_1^2 + T_2^2} \text{ and } \omega^2 = \omega_1^2 + \omega_2^2$$



### Spring in series connection:-

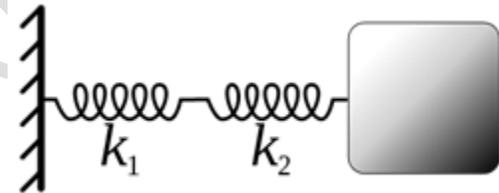
(a) Total spring constant,  $1/k = 1/k_1 + 1/k_2$  or,  
 $k = k_1 k_2 / (k_1 + k_2)$

(b) Time period,  $T^2 = T_1^2 + T_2^2$

(c)  $T = 2\pi\sqrt{m(k_1+k_2)/k_1 k_2}$

(d)  $1/\omega^2 = 1/\omega_1^2 + 1/\omega_2^2$

(e)  $f = 1/2\pi\sqrt{k_1 k_2 / m(k_1 + k_2)}$



### Law's of simple pendulum:-?

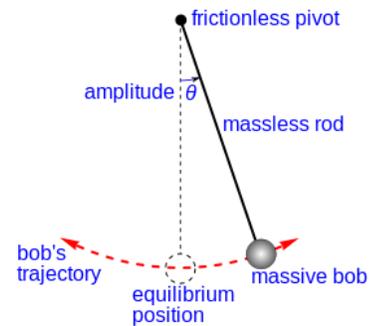
Laws of isochronisms:- Its states that ( $\leq 4^\circ$ ), the time period of a simple pendulum is independent of its amplitude.

Laws of length:- It states that time period of a simple pendulum varies directly as the square root of its length.

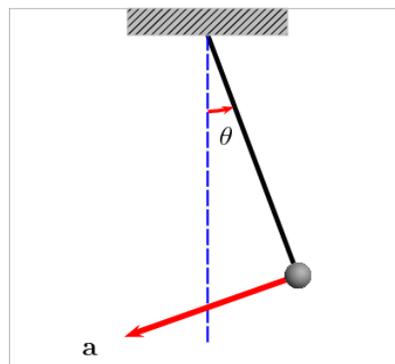
$$T \propto \sqrt{l}$$

Law of acceleration due to gravity:- It states that, the time period of a simple pendulum varies inversely as the square root of acceleration due to gravity at that place.

$$T \propto 1/\sqrt{g}$$



So, Time period of simple pendulum,  $T = 2\pi\sqrt{l/g}$



(a) When placed inside a lift being accelerated upwards, the effective value of  $g$  increases. Thus the time period of pendulum decreases.

(b) When placed inside a lift being accelerated downwards, the effective value of  $g$  decreases. Thus the time period of pendulum increases.

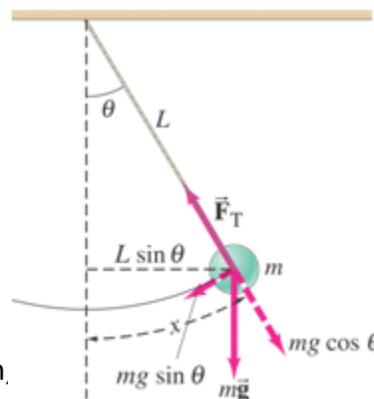
(c) Time period of the pendulum increases at higher altitudes due to decrease in  $g$ .

(d) Time period of the pendulum at a place below the surface of earth decrease due to increase in  $g$ .

(e) At the center of earth ( $g=0$ ). So the time period is infinite.

(f) Time period is greater at equator than at poles.

(g) Due to decrease in the value of  $g$  due to rotation of earth, pendulum increases as the earth rotates about its axis.



(h) **Equation of motion:**  $-d^2\theta/dt^2 + (g/l)\theta = 0$

(i) **Frequency,**  $f = 1/2\pi \sqrt{g/l}$

(j) **Angular frequency,**  $\omega = \sqrt{g/l}$

**Second Pendulum:**-A second's pendulum is that pendulum whose time period is two seconds.

(a)  $T = 2 \text{ sec}$

(b)  $l = 0.9925 \text{ m}$

**Mass-less loaded spring in the horizontal alignment:-**

Force,  $F = -kx$

Acceleration,  $a = -kx/m$

Time period,  $T = 2\pi\sqrt{m/k}$

Frequency,  $f = 1/2\pi\sqrt{k/m}$

**Time period of mass-less loaded spring in the vertical alignment:-**

$T = 2\pi\sqrt{m/k}$  and  $T = 2\pi\sqrt{l/g}$

**Time period of bar pendulum:-**

$T = 2\pi\sqrt{I/mgl}$

Here  $I$  is the rotational inertia of the pendulum.

and

$T = 2\pi\sqrt{L/g}$

Here,  $L = (k^2/I) + l$

**Time period of torsion pendulum:-**

(a)  $T = 2\pi\sqrt{I/C}$

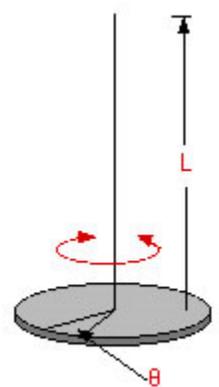
Here  $I$  is the rotational inertia of the pendulum and  $C$  is the restoring couple per unit angular twist.

(b) Equation of motion:  $-d^2\vartheta/dt^2 + (C/I)\vartheta = 0$

Here,  $\vartheta = \vartheta_0 \sin(\omega t + ?)$

(c) Angular frequency,  $\omega = \sqrt{C/I}$

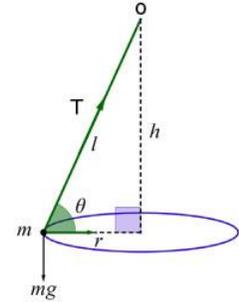
(d) Frequency,  $f = 1/2\pi\sqrt{C/I}$



**Conical Pendulum:-**

Time period,  $T = 2\pi\sqrt{L\cos\theta/g}$

Velocity,  $v = \sqrt{gR\tan\theta}$



**Restoring couple ( $\tau$ ):-**

$\tau = C\theta$

Here C is the restoring couple per unit angular twist and  $\theta$  is the twist produced in the wire.

**Liquid contained in a U-tube:-**

Time period,  $T = 2\pi\sqrt{l/g}$

**Electrical oscillating circuit:-**

Time period,  $T = 2\pi\sqrt{LC}$

Here, L is the inductance and C is the capacitance.

Angular frequency,  $\omega = 1/\sqrt{LC}$

**Ball in a bowl:-**

Time period,  $T = 2\pi\sqrt{(R-r)/g}$

**Free vibrations:-** Vibrations of a body are termed as free vibrations if it vibrates in the absence of any constraint.

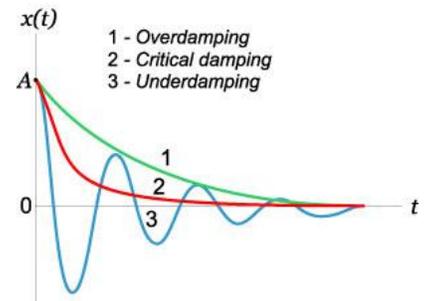
**Damped Vibrations:-**

Equation:  $d^2y/dt^2 + 2\mu dy/dt + \omega^2 y = 0$

Here amplitude,  $R = Ae^{-\mu t}$

And

$\omega' = \sqrt{\omega^2 - \mu^2}$



a)  $\mu \ll \omega$  signifies the body will show oscillatory behavior with gradually decreasing amplitude.

(b)  $\mu \gg \omega$  signifies the amplitude may decrease from maximum to zero without showing the oscillatory behavior.

(c) In between the above two cases, the body is in the state of critically damped.

(d) Time period of oscillation,  $T' = 2\pi/\omega' = 2\pi/\sqrt{\omega^2 - \mu^2}$ . Thus, presence of damping factor  $\mu$  in the denominator indicates an increase of time period due to damping.

**Forced vibrations:-** Forced vibrations is the phenomenon of setting a body into vibrations by a strong periodic force whose frequency is different from natural frequency of body.

**Equation:**  $d^2y/dt^2 + 2\mu dy/dt + \omega^2 y = (F_0/m) \cos pt$

Here,  $\mu = r/2m$  and  $\omega = \sqrt{k/m}$

**Solution:**  $y = A \cos [pt - ?]$

**Amplitude:-**  $A = F_0/m\sqrt{4\mu^2 p^2 + (p^2 - \omega^2)^2}$  and  $A_{\max} = F_0/2\mu m\sqrt{\omega^2 - \mu^2}$

This state of forced vibrations in which the amplitude reaches a maximum value is known as amplitude resonance.

Amplitude vibration depends upon value of  $\omega = \sqrt{k/m}$ . Greater the value of stiffness ( $k$ ), smaller is the amplitude.

**Resonance:-** Resonance is the phenomenon of setting a body into vibrations by a strong periodic force whose frequency coincides with the natural frequency of the body.