- Inertial frame of reference- Reference frame in which Newtonian mechanics holds are called inertial reference frames or inertial frames. Reference frame in which Newtonian mechanics does not hold are called non-inertial reference frames or non-inertial frames.
- The average speed $v_{av}$ and average velocity $\vec{V}_{av}$ of a body during a time interval ?*t* is defined as, $v_{av}$= average speed = ?*s*/?*t*

$$\vec{V}_{av} = \text{average velocity} = \frac{\Delta \vec{r}}{\Delta t}$$

- Instantaneous speed and velocity are defined at a particular instant and are given by

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \text{ and } \vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

**Note:**

a) A change in either speed or direction of motion results in a change in velocity

(b) A particle which completes one revolution, along a circular path, with uniform speed is said to possess zero velocity and non-zero speed.

(c) It is not possible for a particle to possess zero speed with a non-zero velocity.

- Average acceleration is defined as the change in velocity $\Delta\vec{V}$ over a time interval?*t*.

$$\vec{a}_{av} = \frac{\Delta \vec{V}}{\Delta t}$$

The instantaneous acceleration of a particle is the rate at which its velocity is changing at that instant.

$$\vec{a}_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$$

- The three equations of motion for an object with constant acceleration are given below.

(a) v= u+at
(b) s= ut+1/2 $at^2$
(c) $v^2$=$u^2$+2as
Here *u* is the initial velocity, *v* is the final velocity, *a* is the acceleration, *s* is the displacement travelled by the body and *t* is the time.

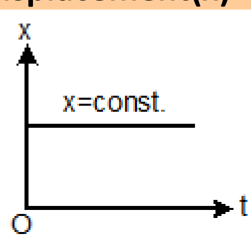
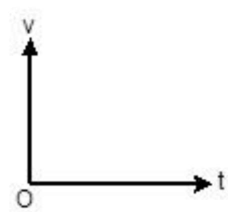
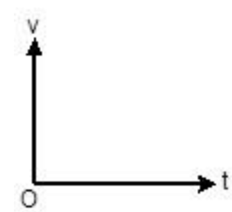
Note: Take '+ve' sign for a when the body accelerates and takes '–ve' sign when the body decelerates.

- The displacement by the body in $n^{th}$ second is given by,

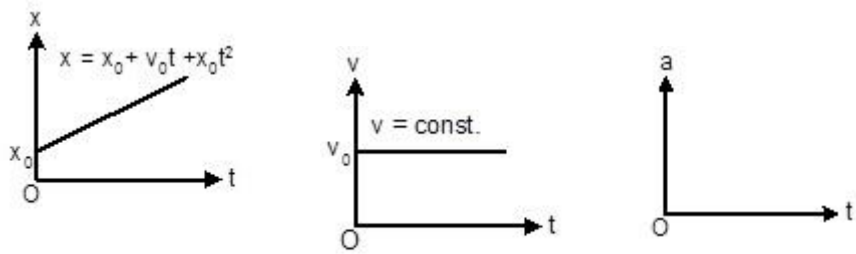
  $s_n$= *u* + *a*/2 (2*n*-1)

- Position-time (*x* vs *t*), velocity-time (*v* vs *t*) and acceleration-time (*a* vs *t*) graph for motion in one-dimension:

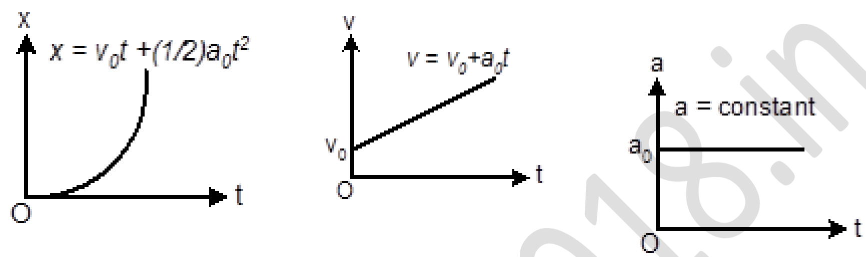
**(i) Variation of displacement (x), velocity (v) and acceleration (a) with respect to time for different types of motion.**

| | Displacement(x) | Velocity(v) | Acceleration (a) |
|---|---|---|---|
| (a) At rest |  |  |  |

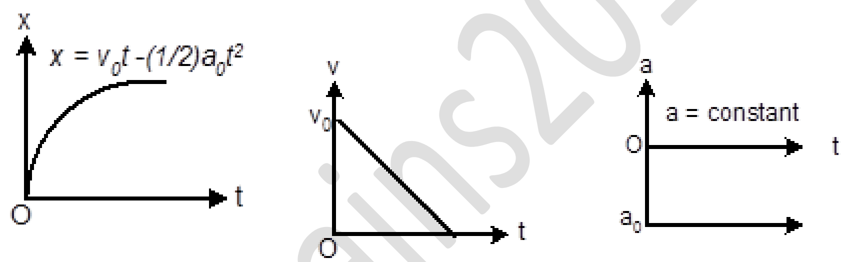
(b) Motion with constant velocity




(c) Motion with constant acceleration




(d) Motion with constant deceleration




- **Scalar Quantities:-** Scalar quantities are those quantities which require only magnitude for their complete specification.(e.g-mass, length, volume, density)
- **Vector Quantities:-** Vector quantities are those quantities which require magnitude as well as direction for their complete specification. (e.g-displacement, velocity, acceleration, force)
- **Null Vector (Zero Vectors):-** It is a vector having zero magnitude and an arbitrary direction.
- When a null vector is added or subtracted from a given vector the resultant vector is same as the given vector.
- Dot product of a null vector with any arbitrary is always zero. Cross product of a null vector with any other vector is also a null vector.
- **Collinear vector:-** Vectors having a common line of action are called collinear vector. There are two types.

Parallel vector ($\vartheta$=0°):- Two vectors acting along same direction are called parallel vectors.

Anti-parallel vector ($\vartheta$=180°):-Two vectors which are directed in opposite directions are called anti-parallel vectors.

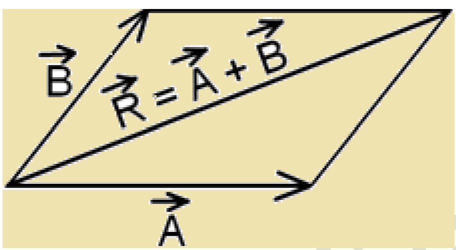
- **Co-planar vectors-** Vectors situated in one plane, irrespective of their directions, are known as co-planar vectors.
- **Vector addition:-**

Vector addition is commutative- $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Vector addition is associative- $\left(\vec{A}+\vec{B}\right)+\vec{C}=\vec{A}+\left(\vec{B}+\vec{C}\right)$

Vector addition is distributive- $m\vec{A}+m\vec{B}=m(\vec{A}+\vec{B})$

**Triangles Law of Vector addition:-** If two vectors are represented by two sides of a triangle, taken in the same order, then their resultant in represented by the third side of the triangle taken in opposite order.




$\vec{R}=\vec{A}+\vec{B}$

Magnitude of resultant vector $\vec{R}$:-

$R=\sqrt{(A^2+B^2+2AB\cos\vartheta)}$

Here $\vartheta$ is the angle between $\vec{A}$ and $\vec{B}$.

If β is the angle between $\vec{R}$ and $\vec{A}$,

then,

$$\beta=\tan^{-1}\{\frac{B\sin\theta}{A+B\cos\theta}\}$$

If three vectors acting simultaneously on a particle can be represented by the three sides of a triangle taken in the same order, then the particle will remain in equilibrium.

So, $\vec{A}+\vec{B}+\vec{C}=0$

**Parallelogram law of vector addition:-**

$$\vec{R}=\vec{A}+\vec{B}$$

$R=\sqrt{(A^2+B^2+2AB\cos\vartheta)}$,
$\beta=\tan^{-1}\{\frac{B\sin\theta}{A+B\cos\theta}\}$

**Cases 1:-** When, $\vartheta=0°$, then,

$R= A+B$ (maximum), $\beta=0°$

**Cases 2:-** When, $\vartheta=180°$, then,

$R= A-B$ (minimum), $\beta=0°$

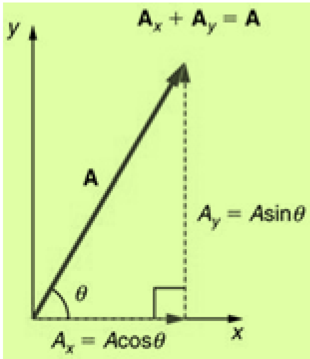
**Cases 3:-** When, $\vartheta=90°$, then,

$R=\sqrt{(A^2+B^2)}$, $\beta = \tan^{-1}(B/A)$

The process of subtracting one vector from another is equivalent to adding, vector ally, the negative of the vector to be subtracted. So,

$$\vec{A}-\vec{B}=\vec{A}+\left(-\vec{B}\right)$$

**Resolution of vector in a plane:-**




- **Product of two vectors:-**

**(a) Dot product or scalar product:-**

$\vec{A}.\vec{B} = AB\cos\theta$ ,

$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zz$

Here A is the magnitude of $\vec{A}$, *B* is the magnitude of $\vec{B}$ and $\vartheta$ is the angle between $\vec{A}$ and $\vec{B}$.

(i) Perpendicular vector:-

$\vec{A}.\vec{B} = 0$

(ii) Collinear vector:-

When, Parallel vector (θ=0°), $\vec{A}.\vec{B} = AB$

When, Anti parallel vector (θ=180°), $\vec{A}.\vec{B} = -AB$

**(b) Cross product or Vector product:-**

$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ Or,

$$\vec{A}\times\vec{B} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{pmatrix}$$

Here *A* is the magnitude of $\vec{A}$, *B* is the magnitude of $\vec{B}$,$\vartheta$ is the angle between $\vec{A}$ and $\vec{B}$ and $\hat{n}$ is the unit vector in a direction perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

(i) Perpendicular vector (θ=90°):-

$$\vec{A}\times\vec{B} = AB$$

(ii) Collinear vector:-

When, Parallel vector (θ=0°), $\vec{A}\times\vec{B} = 0$ (null vector)

When, θ=180°, $\vec{A}\times\vec{B} = 0$ (null vector)

**Unit Vector-** Unit vector of any vector is a vector having a unit magnitude, drawn in the direction of the given vector.
In three dimension,

$$\hat{A} = \frac{\vec{A}}{A}$$
$$= \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

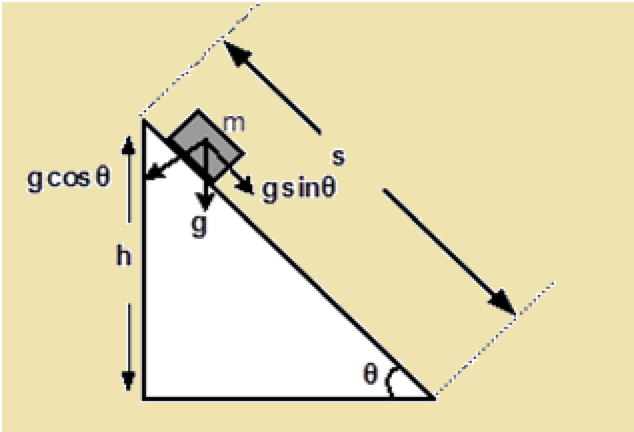
- **Area:-**

Area of triangle:- $A = \frac{1}{2}\left|\vec{A}\times\vec{B}\right|$

Area of parallelogram:- $A = \left|\vec{A}\times\vec{B}\right|$

Volume of parallelepiped:- $\vec{V} = \vec{A}.\left(\vec{B}\times\vec{C}\right)$

**Equation of Motion in an Inclined Plane:**




(i) Perpendicular vector :- At the top of the inclined plane ($t$ = 0, $u$ = 0 and $a$ = $g$ sin$q$ ), the equation of motion will be,

(a) v= (g sinθ)t

(b) $s = ½ (g \sin\theta) t^2$

(c) $v^2 = 2(g \sin\theta)s$

(ii) If time taken by the body to reach the bottom is $t$, then $s = ½ (g \sin\vartheta)\, t^2$

$t = \sqrt{(2s/g \sin\vartheta)}$

But $\sin\vartheta = h/s$ or $s = h/\sin\vartheta$

So, $t = (1/\sin\vartheta)\sqrt{(2h/g)}$

(iii) The velocity of the body at the bottom

$v=g(\sin\vartheta)t$

$=\sqrt{2gh}$

The relative velocity of object $A$ with respect to object $B$ is given by

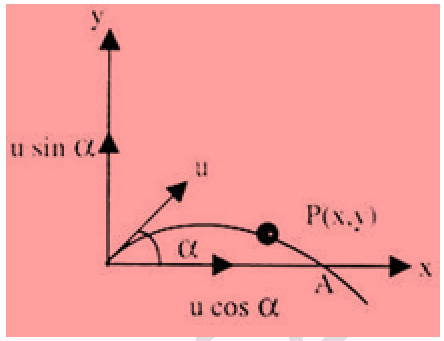
$V_{AB}=V_A-V_B$

Here, $V_B$ is called reference object velocity.

**Variation of mass:-** In accordance to Einstein's mass-variation formula, the relativistic mass of body is defined as, $m = m_0/\sqrt{(1-v^2/c^2)}$

Here, $m_0$ is the rest mass of the body, $v$ is the speed of the body and $c$ is the speed of light.

- **Projectile motion in a plane:-** If a particle having initial speed $u$ is projected at an angle $\vartheta$ (angle of project




Time of Flight, $T = (2u \sin\alpha)/g$

Horizontal Range, $R = u^2 \sin 2\alpha/g$

Maximum Height, $H = u^2 \sin^2\alpha/2g$

Equation of trajectory, $y = x\tan\alpha - (gx^2/2u^2\cos^2\alpha)$

- **Motion of a ball:-**

(a) When dropped:- Time period, $t=\sqrt{(2h/g)}$ and speed, $v=\sqrt{(2gh}$

(b) When thrown up:- Time period, $t=u/g$ and height, $h = u^2/2g$

- **Condition of equilibrium:-**

(a) $\vec{F}_3 = -\left(\vec{F}_1 + \vec{F}_2\right)$

(b) $|F_1+F_2| \geq |F_3| \geq |F_1-F_2|$