

- Let a, b and c form an H.P. Then  $1/a$ ,  $1/b$  and  $1/c$  form an A.P.
- If a, b and c are in H.P. then  $2/b = 1/a + 1/c$ , which can be simplified as  $b = 2ac/(a+c)$
- If 'a' and 'b' are two non-zero numbers then the sequence a, H, b is a H.P.
- The n numbers  $H_1, H_2, \dots, H_n$  are said to be harmonic means between a and b, if a,  $H_1, H_2, \dots, H_n, b$  are in H.P. i.e. if  $1/a, 1/H_1, 1/H_2, \dots, 1/H_n, 1/b$  are in A.P. Let d be the common difference of the A.P., Then  $1/b = 1/a + (n+1)d \Rightarrow d = (a-b)/(n+1)ab$ . Thus  $1/H_1 = 1/a + a-b/(n+1)ab$ ,
  - $1/H_2 = 1/a + 2(a-b)/(n+1)ab$ ,
  - $1/H_n = 1/a + n(a-b)/(n+1)ab$ .

- If  $x_1, x_2, \dots, x_n$  are n non-zero numbers, then the harmonic mean 'H' of these numbers is given by  $1/H = 1/n (1/x_1 + 1/x_2 + \dots + 1/x_n)$

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^n \frac{1}{x_i}} = \frac{n \cdot \prod_{j=1}^n x_j}{\sum_{i=1}^n \frac{\prod_{j=1}^n x_j}{x_i}}$$

- As the nth term of an A.P is given by  $a_n = a + (n-1)d$ , So the nth term of an H.P is given by  $1/[a + (n-1)d]$ .
- If we have a set of weights  $w_1, w_2, \dots, w_n$  associated with the set of values  $x_1, x_2, \dots, x_n$ , then the weighted harmonic mean is defined as

$$\frac{\sum_{i=1}^n w_i}{\sum_{i=1}^n \frac{w_i}{x_i}}$$

- Questions on Harmonic Progression are generally solved by first converting them into those of Arithmetic Progression.
- If 'a' and 'b' are two positive real numbers then  $A.M \times H.M = G.M^2$
- The relation between the three means is defined as  $A.M > G.M > H.M$
- If we need to find three numbers in a H.P. then they should be assumed as  $1/a-d, 1/a, 1/a+d$
- Four convenient numbers in H.P. are  $1/a-3d, 1/a-d, 1/a+d, 1/a+3d$
- Five convenient numbers in H.P. are  $1/a-2d, 1/a-d, 1/a, 1/a+d, 1/a+2d$