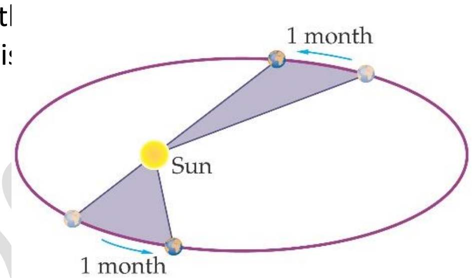
## Gravitation:-

**Kepler's first law (law of elliptical orbit):-** A planet moves round the sun in an elliptical orbit with sun situated at one of its foci.

**Kepler's second law (law of ars**

**eal velocities):-** A planet moves round the sun in such a way that its areal velocity is constant.

**Kepler's third law (law of time period):-** A planet moves round tl square of its period is proportional to the cube of semi major axi:

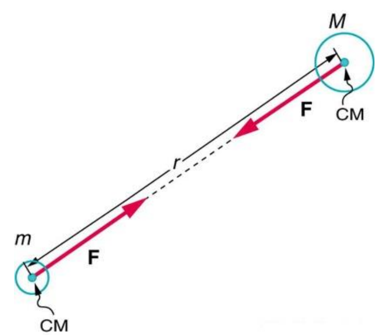



$T^2 \propto R^3$

Here R is the radius of orbit.

$T^2 = (4\pi^2/GM)R^3$

**Newton's law of gravitation:-**




Every particle of matter in this universe attracts

every other particle with forcer which varies

directly as the product of masses of two particles

and inversely as the square of the distance between them.

$F = GMm/r^2$

Here, $G$ is universal gravitational constant. $G = 6.67 ´10^{-11}$ $Nm^2 / kg^2$

**Dimensional formula of G:** $G = Fr^2/Mm = [MLT^{-2}][L^2]/[M^2] = [M^{-1}L^3T^{-2}]$

**Acceleration due to gravity (g):-** $g = GM/R^2$

**Variation of g with altitude:-** $g' = g(1- 2h/R)$, if $h<<R$. Here R is the radius of earth and h is the height of the body above the surface of earth.

**Variation of g with depth:-** $g' = g(1- d/R)$. Here g' be the value of acceleration due to gravity at the depth d.

**Variation with latitude:-**

At poles:- $\vartheta = 90^\circ$, $g' = g$

At equator:- $\vartheta = 0^\circ$, $g' = g\,(1-\omega^2 R/g)$

Here $\omega$ is the angular velocity.

As $g = GM_e/R_e^2$ , therefore $g_{pole} > g_{equator}$

Gravitational Mass: - $m = FR^2/GM$

Gravitational field intensity:-

$E = F/m$

$= GM/r^2$

**Weight:-** $W= mg$

**Gravitational intensity on the surface of earth ($E_s$):-**

$= 4/3\,(\pi R\rho G)$

Here R is the radius of earth, ρ is the density of earth and G is the gravitational constant.

**Gravitational potential energy (U):-** $U = -GMm/r$

(a) Two particles: $U = -Gm_1m_2/r$

(b) Three particles: $U = -Gm_1m_2/r_{12} - Gm_1m_3/r_{13} - Gm_2m_3/r_{23}$

**Gravitational potential (V):-** $V(r) = -GM/r$

At surface of earth,

$V_s = -GM/R$

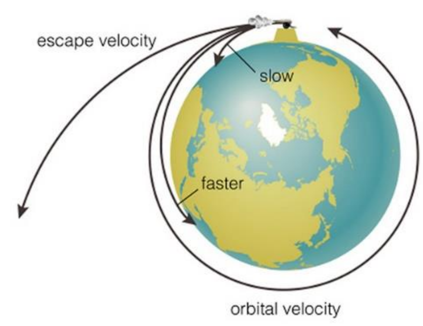
Here R is the radius of earth.

**Escape velocity ($v_e$):-** It is defined as the least

Velocity with which a body must be projected Vertically

upward in order that it may just escape the gravitational

pull of earth.




$v_e = \sqrt{2GM/R}$

or, $v_e = \sqrt{2gR} = \sqrt{gD}$

Here $R$ is the radius of earth and $D$ is the diameter of the earth.

**Escape velocity ($v_e$) in terms of earth's density:-** $v_e = R\sqrt{8\pi G\rho/3}$

**Orbital velocity ($v_0$):-**

$v_0 = \sqrt{GM/r}$

If a satellite of mass m revolves in a circular orbit around the earth of radius R and h be the height of the satellite above the surface of the earth, then,

$r = R+h$

So, $v_0 = \sqrt{MG/R+h} = R\sqrt{g/R+h}$

In the case of satellite, orbiting very close to the surface of earth, then orbital velocity will be,

$v_0 = \sqrt{gR}$

**Relation between escape velocity ve and orbital velocity $v_0$ :-** $v_0 = v_e/\sqrt{2}$ (if $h \ll R$)

**Time period of Satellite:-** Time period of a satellite is the time taken by the satellite to complete one revolution around the earth.

$T = 2\pi\sqrt{(R+h)^3/GM} = (2\pi/R)\sqrt{(R+h)^3/g}$

If $h \ll R$, $T = 2\pi\sqrt{R/g}$

**Height of satellite:-** $h = [gR^2T^2/4\pi^2]1/3 – R$

**Energy of satellite:-**

Kinetic energy, $K = ½\, mv_0^2 = ½\, (GMm/r)$

Potential energy, $U = -\, GMm/r$

Total energy, $E = K+U$

$= ½\, (GMm/r) + (-\, GMm/r)$

$= -½\, (GMm/r)$

**Gravitational force in terms of potential energy:- F = – (dU/dR)**

**Acceleration on moon:-**

$g_m = GM_m/R_m^2 = 1/6\ g_{earth}$

Here $M_m$ is the mass of moon and $R_m$ is the radius of moon.

**Gravitational field:-**

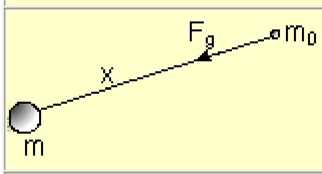
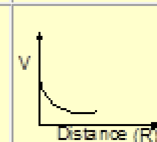
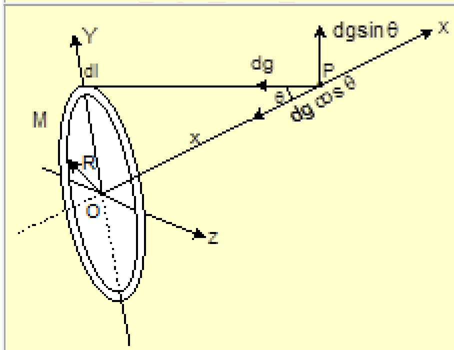
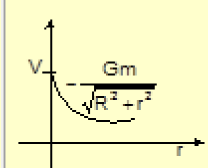
(a) Inside:-

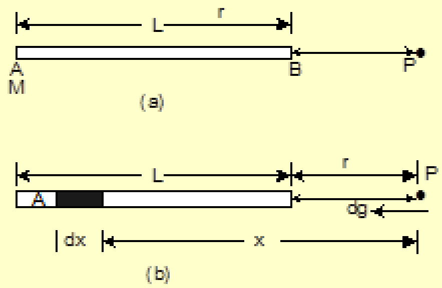
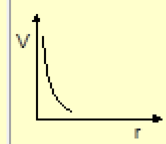
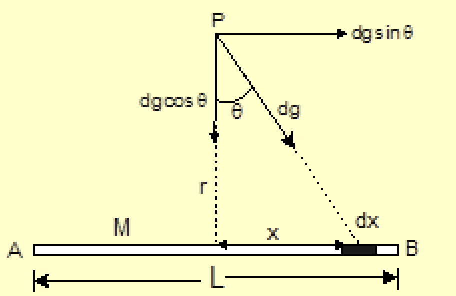
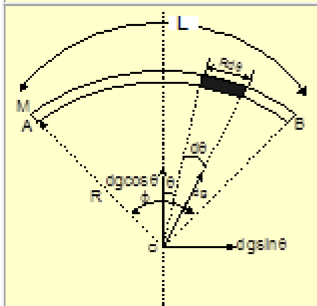
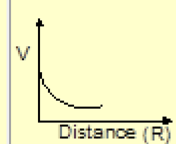
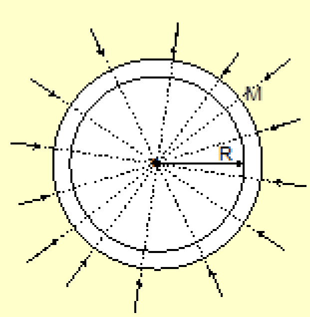
$$\vec{g} = -\frac{GM}{R^3}r\hat{r}$$

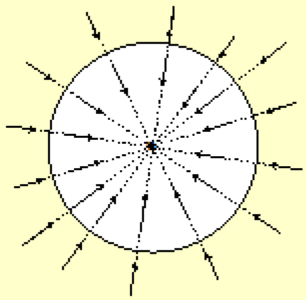
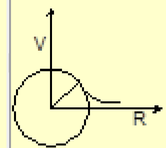
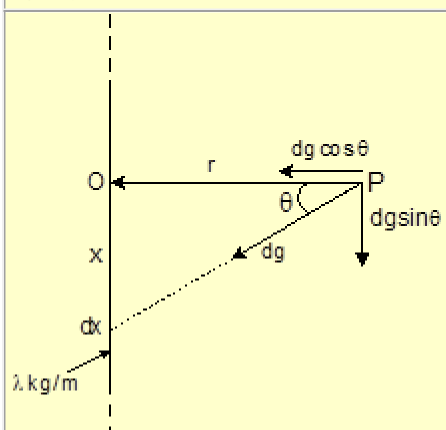
(b) Outside:-

$$\vec{g} = -\frac{GM}{r^2}\hat{r}$$

**GRAVITATIONAL POTENTIAL & FIELD DUE TO VARIOUS OBJECTS**

| Causing Shape | Gravitational Potential (V) | Gravitational Field (I or E) | Graph<br>V vs R |
|---|---|---|---|
| **POINT MASS** | | | |
|   | $V = \frac{-GM}{R}$ | $I \text{ or } E = \frac{GM}{R^2}$ |   |
| **AT A POINT ON THE AXIS OF RING** | | | |
|   | $V = \frac{-GMr}{\sqrt{R^2 + r^2}}$<br>$0 \le r \le \infty$ | $E(r) = \frac{-GMr}{(R^2 + r^2)^{3/2}}$ |   |
| **ROD** | | | |
| **1. AT AN AXIAL POINT** | | | |

| | | | |
|---|---|---|---|
|   | $V = -\frac{GM}{L}\ln\left(1+\frac{L}{r}\right)$ | $E = \frac{GM}{r^2}\left(\frac{1}{1+\frac{L}{r}}\right)$ |   |
| **2. AT AN EQUATORIAL POINT** | | | |
|   | $V = \frac{2GM}{r\sqrt{l^2+4r^2}}$ | $E = -\frac{dv}{dr}$ | |
| **CIRCULAR ARC** | | | |
|   | $V = \frac{2\pi GM}{L}$ | $E = \frac{2\pi GM}{L^2}$ |   |
| **HOLLOW SPHERE** | | | |
|   | $V(r) = \frac{-GM}{r}$ $(r \geq R)$ $V(r) = \frac{-GM}{R}$ $(r = R)$ | $E(r) = \frac{GM}{r^2}$ $(r \geq R)$ $E(r) = \frac{GM}{R^2}$ $(r = R)$ | |

| | | | |
|---|---|---|---|
| **SOLID SPHERE** | | | |
|  | $V(r) = \frac{-GM}{r}$ $(r \geq R)$ $V(r) = \frac{-GM}{R^3}(1.5R^3 - 1.5R^2)$ $(r \leq R)$ | $E(r) = \frac{GM}{r^2}$ $E(r) = \frac{GMr}{R^3}$ $(r \leq R)$ |  |
| **LONG THREAD** | | | |
|  | $V = \infty$ | $E = \frac{2Gl}{r}$ | |

## Projectile:-

**Projectile fired at angle α with the horizontal:-** If a particle having initial speed u is projected at an angle α (angle of projection) with x-axis, then,

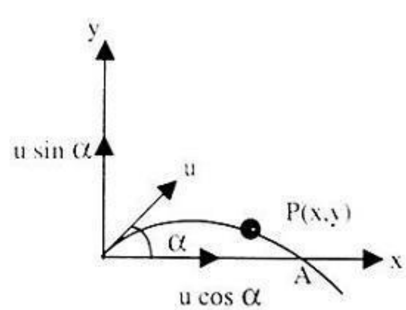
**Time of Ascent,** $t = (u \sin\alpha)/g$

**Total time of Flight,** $T = (2u \sin\alpha)/g$

**Horizontal Range,** $R = u^2 \sin 2\alpha/g$

**Maximum Height,** $H = u^2 \sin^2\alpha/2g$

**Equation of trajectory,** $y = x\tan\alpha - (gx^2/2u^2\cos^2\alpha)$



**Instantaneous velocity,** $V = \sqrt{(u^2 + g^2t^2 - 2ugt \sin\alpha)}$

and

$\beta = \tan^{-1}(u\sin\alpha - gt/u\cos\alpha)$

**Projectile fired horizontally from a certain height:-**

**Equation of trajectory:** $x^2 = (2u^2/g)y$

**Time of descent** (timer taken by the projectile to come down to the surface of earth), $T = \sqrt{2h/g}$

**Horizontal Range,** $H = u\sqrt{2h/g}$. Here $u$ is the initial velocity of the body in horizontal direction.

**Instantaneous velocity**:-

$V = \sqrt{u^2 + g^2t^2}$

If $\beta$ be the angle which $V$ makes with the horizontal, then,

$\beta = \tan^{-1}(-gt/u)$

**Projectile fired at angle α with the vertical:-**

**Time of Ascent,** $t = (u \cos\alpha)/g$

**Total time of Flight,** $T = (2u \cos\alpha)/g$

**Horizontal Range,** $R = u^2\sin 2\alpha/g$

**Maximum Height,** $H = u^2\cos^2\alpha/2g$

**Equation of trajectory,** $y = x \cot\alpha - (gx^2/2u^2\sin^2\alpha)$

**Instantaneous velocity,** $V = \sqrt{(u^2 + g^2t^2 - 2ugt \cos\alpha)}$ and $\beta = \tan^{-1}(u\cos\alpha - gt/u\sin\alpha)$

**Projectile fired from the base of an inclined plane:-**

**Horizontal Range**, $R = 2u^2 \cos(\alpha+\beta) \sin\beta/g\cos^2\alpha$

**Time of flight,** $T = 2u \sin\beta/ g\cos\alpha$

Here, $\alpha+\beta=\theta$