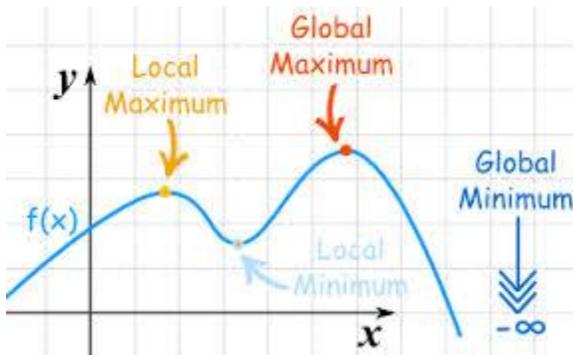


- The concepts of straight line, maxima and minima, global maxima and minima, Rolle's Theorem and LMVT all come under the head of Application of Derivatives.
- If a function is increasing on some interval then the slope of the tangent is positive at every point of that interval due to which its derivative is positive.
- Similarly, the derivative of a function which is decreasing on some interval is negative as the slope of the tangent is negative at every point of that interval.



- A function f is said to have a local maximum (also termed as relative maximum) at $x = a$ if $f(x) \leq f(c)$, for every x in some open interval around $x = c$.
- A function f is said to have a relative minimum or a local minimum around $x = c$ if $f(x) \geq f(c)$, for every x in some open interval around $x = a$.
- A function f is said to have a global maximum (also termed as absolute maximum) at $x = a$ if $f(x) \leq f(c)$, for every x in the domain under consideration.
- A function f is said to have an absolute minimum or a global minimum around $x = c$ if $f(x) \geq f(c)$, for every x in the whole domain under consideration.

Rolle's Theorem

Let $y = f(x)$ be a given function which satisfies the conditions:

- 1) $f(x)$ is continuous in $[a, b]$
- 2) $f(x)$ is differentiable in (a, b)

$$3) f(a) = f(b)$$

Then $f'(x) = 0$ at least once for some $x \in (a, b)$.

- Certain points to be noted in Rolle's Theorem include:
 - Converse of the theorem does not hold good.
 - There can be more than one such c .
 - The conditions of Rolle's Theorem are only sufficient and not necessary.

Lagrange Mean Value Theorem (LMVT)

- If a given function $y = f(x)$ satisfies certain conditions like:

$f(x)$ is continuous in $[a, b]$

$f(x)$ is differential in (a, b)

- then $f'(x) = [f(b) - f(a)]/[b-a]$ for some $x \in (a, b)$. This is the generalization of the Rolle's Theorem and is termed as Lagrange Mean Value theorem.
- A function is said to be monotonically increasing at $x = a$ if $f(x)$ satisfies $f(a+h) > f(a)$ and $f(a-h) < f(a)$, for some small positive h .
- A function is said to be monotonically decreasing at $x = a$ if $f(x)$ satisfies $f(a+h) < f(a)$ and $f(a-h) > f(a)$, for some small positive h .
- If $f'(x) > 0 \forall x \in (a, b)$ and points which make equal to zero (in between (a, b)) don't form an interval, then $f(x)$ would be increasing in $[a, b]$ otherwise it will be non-decreasing function.
- If $f'(x) < 0 \forall x \in (a, b)$ and points which make equal to zero (in between (a, b)) don't form an interval, $f(x)$ would be decreasing in $[a, b]$, otherwise it will be non-increasing.
- For all x and y , such that $x \leq y$, if $f(x) \leq f(y)$, then the function f is said to be monotonically increasing, increasing or non-decreasing.
- Similarly, for $x \leq y$, if $f(x) \geq f(y)$, then the function is monotonically decreasing, decreasing or non-increasing i.e. it reverses the order.
- If f is increasing for $x > a$ and f is also increasing for $x < a$ then f is also increasing at $x = a$ provided $f(x)$ is continuous at $x = a$.
- If $f(x)$ is strictly increasing, then f^{-1} exists and is also strictly increasing.

- If $f(x)$ is strictly increasing on $[a, b]$ and is also continuous then f^{-1} is continuous on $[f(a), f(b)]$.
- If $f(x)$ and $g(x)$ are strictly increasing (decreasing) functions on $[a, b]$, then $g \circ f(x)$ is strictly increasing (decreasing) function on $[a, b]$.
- If one of the two functions $f(x)$ and $g(x)$ is strictly increasing and other is strictly decreasing then $g \circ f(x)$ is strictly decreasing on $[a, b]$.
- If a continuous function $y = f(x)$ is strictly increasing in the closed interval $[a, b]$, then $f(a)$ is the least value.
- If $f(x)$ is decreasing in $[a, b]$, then $f(b)$ is the least and $f(a)$ is the greatest value of $f(x)$ in $[a, b]$.
- If $f(x)$ is non-monotonic in $[a, b]$ and is continuous then the greatest and the least value of $f(x)$ in $[a, b]$ are those where $f'(x) = 0$ or $f'(x)$ does not exist or at the extreme values.
- The direction of acceleration is in the direction of velocity or opposite to it.
- When the particle is going upward, the value of g is negative and when it is coming back, the value of g is positive.
- At maximum height the velocity of a particle is zero. The value of g is 9.8 m/s^2 or 980 cm/s^2 .
- Slope of tangent to the curve $y = f(x)$ at the point (x, y) is $m = \tan \theta = \left[\frac{dy}{dx} \right]_{(x,y)}$
- If the equation of the curve is in the parametric form $x = f(t)$ and $y = g(t)$, then the equations of the tangent and the normal are $y - g(t) = g'(t)/f'(t)(x - f(t))$ and $f'(t)[x - f(t)] + g'(t)[y - g(t)] = 0$ respectively.
- The equation of tangent to the curve $y = f(x)$ at the point $P(x_1, y_1)$ is given by $y - y_1 = \left[\frac{dy}{dx} \right]_{(x,y)} (x - x_1)$
- If $dy/dx = 0$ then the tangent to curve $y = f(x)$ at the point (x, y) is parallel to the x -axis.
- If $dy/dx \rightarrow \infty$, $dx/dy = 0$, then the tangent to the curve $y = f(x)$ at the point (x, y) is parallel to the y -axis.
- If $dy/dx = \tan \theta > 0$, then the tangent to the curve $y = f(x)$ at the point (x, y) makes an acute angle with positive x -axis and vice versa.
- If two curves are orthogonal, then the product of their slopes is -1 everywhere wherever they intersect.

Length of tangent, normal, subtangent, subnormal:

$$\text{Tangent} = \left| \frac{y\sqrt{1 + (dy/dx)^2}}{dy/dx} \right|$$

$$\text{Subtangent} = \left| \frac{y}{dy/dx} \right|$$

$$\text{Normal} = \left| y\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \right|$$

$$\text{Subnormal} = \left| y \left(\frac{dy}{dx}\right) \right|$$